



Welcome to

# RAMOO 2024

11th Workshop on Recent Advances  
in Multi-Objective Optimization

September 12-13, 2024

University of Wuppertal  
Optimization Group



BERGISCHE  
UNIVERSITÄT  
WUPPERTAL



## **Local Organizers**

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# RAMOO 2024

## **Workshop Program**

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# Day 1

8:30 a.m. - 9:00 a.m.

Registration

9:00 a.m. - 9:30 a.m.

Opening Session

9:30 a.m. - 10:30 a.m.

**Keynote Lecture I**

*Upper bound sets for bi-objective knapsack problems*

Anthony Przybylski, Nantes Université

10:30 a.m. - 11:00 a.m.

**Invited Talk**

*The parallel epsilon algorithm for tri-objective integer optimization problems*

Kathrin Prinz, RPTU

11:00 a.m. - 11:30 a.m.

Coffee Break

11:30 a.m. - 1:00 p.m.

**Invited Talks**

*Finding quasi-cliques: a multiobjective perspective*

Daniela Scherer dos Santos, University of Coimbra

*Labeling methods for partially ordered paths*

Ricardo Euler, ZIB

*Consensus-based optimization for multi-objective problems*

Claudia Totzeck, Wuppertal University

1:00 p.m. - 2:00 p.m.

Group Photo & Lunch

2:00 p.m. - 3:30 p.m.

**Invited Talks**

*Effective front-descent algorithms with convergence guarantees*

Davide Pucci, University of Florence

*Sensitivity analysis of the cost coefficients in multi-objective integer linear optimization*

Nicolas Forget, Linz University

*Asymptotic bounds for the multiobjective shortest path problem*

Pedro Maristany de las Casas, ZIB

3:30 p.m. - 4:00 p.m.

Coffee Break

4:00 p.m. - 5:00 p.m.

**Invited Talks**

*Supported nondominated points as a representation of the nondominated set: An empirical analysis*

Serpil Sayin, Koç University

*Multi-stage multi-objective optimization*

Andreas Löhne, Friedrich-Schiller-Universität Jena

5:00 p.m. - 6:00 p.m.

**Keynote Lecture II**

*Robust multi-objective optimization*

Anita Schöbel, RPTU & ITWM

8:00 p.m.

Conference Dinner

# Day 2

9:00 a.m. - 10:30 a.m.

## **Tutorial I**

*Branch and bound for multi-objective optimization: overview and future challenges*

Sune Lauth Gadegaard, Aarhus University

10:30 a.m. - 11:00 a.m.

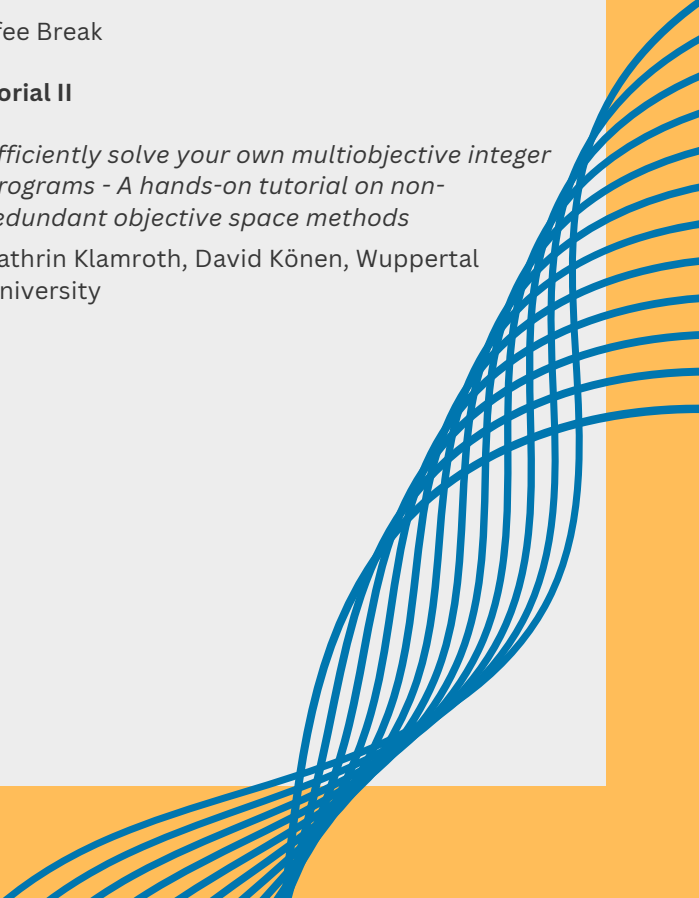
Coffee Break

11:00 a.m. - 12:30 a.m.

## **Tutorial II**

*Efficiently solve your own multiobjective integer programs - A hands-on tutorial on non-redundant objective space methods*

Kathrin Klamroth, David Könen, Wuppertal University





# Abstracts

Thursday, September 12, 9:30 a.m. - 10:30 a.m.

## ***Keynote Lecture I: Upper bound sets for bi-objective knapsack problems***

Anthony Przybylski, Nantes Université

Bound sets are a key point to reduce the necessary enumeration to solve exactly multi-objective combinatorial optimization problems. In practice, the quality of bound sets is as important as their computational cost. A good trade-off must therefore be found. To achieve this goal, the problem structure can be used.

In this study, we consider upper bound sets for two variants of bi-objective knapsack problems in binary variables. For the uni-dimensional case, we show that it is possible to preprocess the computation of the linear relaxation, reducing drastically its computational cost. Next, we consider a problem-specific bound set that extends the well-known Martello and Toth's bound to the bi-objective case. For the bi-dimensional case, we consider the surrogate relaxation. The exact solution of the dual surrogate problem provides a good upper bound set but for a large computational cost, we consider therefore an approximate solution of this problem.

Finally, the impact of the proposed upper bound sets is tested in enumerative solution methods.



Thursday, September 12, 10:30 a.m. - 11:00 a.m.

***Invited Talk: The parallel epsilon algorithm for tri-objective integer optimization problems***

Kathrin Prinz, Stefan Ruzika, University of Kaiserslautern-Landau

We present a new algorithm, the Parallel Enumeration Algorithm (PEA), for enumerating the nondominated set of tri-objective integer optimization problems. PEA only requires solving a linear number of lexicographic epsilon-constraint scalarization problems. Furthermore, PEA is easy to implement and easy to parallelize: A novel order on the nondominated set, induced by the structure of the parameter space of the lexicographic epsilon-constraint scalarization is utilized to split the computation into a number of independent parallel tasks, such that no communication between different tasks is required. Consequently, the communication overhead, one of the main factors of diminishing speedups, is minimal. Additionally, we discuss the potential extension of PEA to problems with more than three objectives.

The performance of the algorithm was evaluated and compared to the CPLEX parallelization of a state-of-the-art sequential algorithm for tri-objective integer optimization problems on benchmark knapsack and assignment problems for up to 16 threads. Additionally, we investigated the scaling of PEA with up to 128 threads. The results of the computational study demonstrate the effectiveness of the proposed algorithm and the computational advantage of its parallelization, as it achieves an almost linear speed-up in the number of threads. Furthermore, the new algorithm outperforms the CPLEX parallelization of the state-of-the-art algorithm as soon as more than two threads are available.

Thursday, September 12, 11:30 a.m. - 12:00 p.m.

### ***Invited Talk: Finding quasi-cliques: a multiobjective perspective***

Daniela Scherer dos Santos, University of Coimbra, Kathrin Klamroth, Wuppertal University, Pedro Martins, University of Lisbon, Luís Paquete, University of Coimbra

In a simple undirected graph  $G$ , a quasi-clique is defined as a subgraph whose density is at least  $\gamma$  ( $0 < \gamma \leq 1$ ). Two optimization problem variants can be defined for quasi-cliques: one seeks to maximize vertex cardinality given a fixed density, while the other aims to maximize density given a fixed cardinality. However, when no a priori preference information about cardinality and density is available, a more natural approach is to consider the problem from a multiobjective perspective. We introduce the Multiobjective Quasi-clique Problem (MOQC), which aims to find a quasi-clique by simultaneously maximizing both vertex cardinality and edge density. To efficiently address this problem, we explore its key properties as well as those of quasi-cliques. Furthermore, we analyze the connections between the MOQC problem, its single-objective counterparts, and a biobjective optimization problem. We propose methods based on  $\epsilon$ -constraint, dichotomic search, and local search strategies. Our presentation will cover the proposed methodologies and share the experimental results on a wide set of real-life instances.

Acknowledgments: Daniela Scherer dos Santos acknowledges the Foundation for Science and Technology (FCT) for the Ph.D. fellowship 2022.12082.BD. This work is partially funded by the FCT, I.P./MCTES through national funds (PIDDAC), within the scope of CISUC R&D Unit – UID/CEC/00326/2020.

Thursday, September 12, 12:00 p.m. - 12:30 p.m.

### ***Labeling methods for partially ordered paths***

Ricardo Euler, The Zuse Institute Berlin

The multi-objective shortest path problem (MOSP) asks for a set of efficient paths with respect to multiple weight functions. This induces a specific partial order on the path set, namely the  $n$ -fold product order of the canonical order on the reals. It is therefore natural to ask whether methods for MOSP can be extended to more general partial orders. For several specific applications, this question has been answered affirmatively, but a general investigation was previously lacking. In this talk, we first formalize the partial order shortest path problem. It captures the particular structure of these applications as special cases. Examples range from time-dependent shortest path and bottleneck path problems to the electric vehicle shortest path problem with recharging and complex financial weight functions studied in the public transportation community. In this generality, we then study optimality conditions or the lack of them, depending on the objective functions' properties. Our results hold for general digraphs and, therefore, surpass previous results that were limited to acyclic graphs.

Thursday, September 12, 12:30 p.m. - 1:00 p.m.

## ***Consensus-based optimization for multi-objective problems***

Claudia Totzeck, Wuppertal University

in order to bridge the gap between heuristics and provably convergent particle-based global optimization approaches, the Consensus-based optimization method was tailored to be feasible for fast implementation on the one hand, and a PDE-approximation that allows for a rigorous convergence analysis on the other. We discuss the main features of the method and its convergence result for the single-objective case and then extend the method towards multi-objective problems using a multi-swarm approach. Under appropriate assumptions we show that many of the convergence results extend to the multi-objective setting. The theoretical results are underpinned by numerical examples that highlight the simple intuition behind the scheme.

Main references:

- [1] R. Pinnau, C. Totzeck, O. Tse, S. Martin. A consensus-based model for global optimization and its mean-field limit, *Mathematical Models and Methods in Applied Sciences* 27 (1), pp. 183-204, 2017.
- [2] J. A. Carrillo, Y.-P. Choj, C. Totzeck, O. Tse. An analytical framework for a consensus-based global optimization method, *Mathematical Models and Methods in Applied Sciences* 28 (06), pp. 1037-1066, 2018.
- [3] K. Klamroth, M. Stiglmayr, C. Totzeck. Consensus-based optimization for multi-objective problems: a multi-swarm approach, *Journal of Global Optimization*, 2024.

Thursday, September 12, 2:00 p.m. - 2:30 p.m.

## ***Effective front-descent algorithms with convergence guarantees***

Davide Pucci, University of Florence

In the context of unconstrained continuous multi-objective optimization, we introduce the Front-Descent framework. This class of descent-type methods is designed to reconstruct the Pareto set, and can be considered as a generalization of the Front Steepest Descent algorithm [1] in which suitable, effective search directions (e.g., Newton, Quasi-Newton, Barzilai-Borwein) can be employed. We provide a deep characterization of the behavior and the mechanisms of the algorithmic framework, showing that popular search directions can indeed be soundly used within the procedure. Moreover, we show that, under reasonable assumptions, standard convergence results and some complexity bounds hold. In addition, we introduce a novel type of convergence results, concerning sequence of sets produced by the algorithm. These considerations allow us to define suitable termination conditions for the proposed class of methods. Finally, we report the results from a large experimental benchmark showing that the proposed class of approaches far outperforms state-of-the-art methodologies.

References:

[1] Guido Cocchi, Giampaolo Liuzzi, Stefano Lucidi, and Marco Sciandrone. On the convergence of steepest descent methods for multiobjective optimization. *Computational Optimization and Applications*, 77:1–27, 2020.

Thursday, September 12, 2:30 p.m. - 3:00 p.m.

***Sensitivity analysis of the cost coefficients in multi-objective integer linear optimization***

Nicolas Forget, University Linz

We consider sensitivity analysis of the cost coefficients in multi-objective integer linear programming problems. We define the sensitivity region as the set of simultaneous changes to the coefficients for which the efficient set and its structure remain the same. In particular, we require that the component-wise relation between efficient solutions is preserved and that inefficient solutions remain inefficient, and we show that this is sufficient for the efficient set to be the same upon changes. In this talk, we focus on the case where a single objective coefficient is modified at a time, even though our technique extends to multiple simultaneous changes. We demonstrate that inefficient solutions need to be investigated, but that it suffices to inspect those that are efficient in one of two related  $q + 1$ -objective problems. Computational experiments are carried out on binary and integer knapsack problems to illustrate the general applicability of our technique.

Thursday, September 12, 3:00 p.m. - 3:30 p.m.

## ***Asymptotic bounds for the multiobjective shortest path problem***

Pedro Maristany de las Casas, The Zuse Institute Berlin

Polynomial output-sensitive asymptotic bounds for the One-to-All Multiobjective Shortest Path (MOSP) problem can be derived by analyzing label-setting MOSP algorithms. These algorithms leverage the subpath efficiency principle, expanding only efficient subpaths. Consequently, during a label-setting MOSP algorithm, a path is, at any point in time, classified into one of four categories: unexplored, discarded, permanent, or explored. The first two categories are not explicitly managed. The set of permanent paths, which are efficient paths, constitutes the algorithm's output. Recent advances in fast label-setting MOSP algorithms focus on effectively managing the set of explored paths. Among these algorithms, the new Multiobjective Dijkstra Algorithm achieves the best asymptotic bounds in terms of running time and space consumption. However, it is not always the optimal choice in practice. This talk will explore the current landscape of label-setting MOSP algorithms and examine how recent advancements have also impacted the solvability of other problems where MOSP arises as a subroutine.

Thursday, September 12, 4:00 p.m. - 4:30 p.m.

***Supported nondominated points as a representation of the nondominated set: An empirical analysis***

Serpil Sayin, Koç University

The nondominated set of a Multiple Objective Discrete Optimization (MODO) problem is known to contain unsupported nondominated points, which outnumber the supported ones and are more difficult to obtain. We treat supported nondominated points as a representation and analyze their quality using different metrics beyond their sheer numbers. Under different data generation schemes on multiobjective knapsack and assignment problems, we observe that supported nondominated points almost always provide a good representation of the entire nondominated set.



Thursday, September 12, 4:30 p.m. - 5:00 p.m.

### ***Multi-stage multi-objective optimization***

Andreas Löhne, Friedrich Schiller University Jena

We consider multi-objective optimization problems involving two decision makers. The first decision maker, who acts initially, is responsible for a subset of the variables. The second decision maker, who acts subsequently, is responsible for the remaining variables. Such a multi-stage decision problem can be interpreted as an optimization problem with a set-valued objective function, known as a set optimization problem. We discuss this type of problem, focusing on solution methods, the decision-making process, and applications.

Thursday, September 12, 5:00 p.m. - 6:00 p.m.

## ***Keynote II: Robust multi-objective optimization***

Anita Schöbel, University of Kaiserslautern-Landau and  
Fraunhofer Institute for Industrial Mathematics ITWM

Most real-world optimization problems contain parameters which are not known at the time a decision is to be made. Data may not be measurable in the precision needed, may only be estimated or depend on future developments. An optimal solution which does not take such an uncertainty into account often becomes bad or even infeasible for the scenario which finally realizes. A robust solution tries to hedge against the uncertainty.

Robust optimization provides models and techniques for taking such uncertainties of data into account. We start by sketching robustness concepts for single-objective robust optimization problems showing that already in the scalar case many definitions for a solution being *\emph{robust}* exist. For multi-objective optimization, we have even more possibilities to define what it means that a solution is *\emph{robust (Pareto) efficient}*. We sketch many of these multi-objective robustness concepts. We provide a characterization of such concepts by classifying them as scenario-based, set-based and vector-based concepts. We review some of their properties, discuss their relations and provide a method on how they can be compared.

Finally, we briefly sketch algorithmic approaches and challenges.

Friday, September 13, 9:00 a.m. - 10:30 a.m.

***Tutorial I: Branch and bound for multi-objective optimization: overview and future challenges***

Sune Lauth Gadegaard, Aarhus University

This talk will provide an overview of the current state-of-the-art within branch and bound algorithms for multi-objective integer optimization. The primary focus will be on linear combinatorial optimization problems, though other problem types will also be discussed. Key elements of the algorithms and their impact will be explored, and promising directions for future research will be outlined. The talk will be followed by a tutorial session that will delve into the fundamental components of a multi-objective branch and bound algorithm.

Friday, September 13, 11:00 a.m. - 12:30 p.m.

***Tutorial II: Efficiently solve your own multiobjective integer programs - A hands-on tutorial on non-redundant objective space methods***

Kathrin Klamroth, David Könen, Wuppertal University

In this hands-on tutorial, we present an efficient and easily applicable method for the exact solution of multiobjective integer programming problems (MOIPs) with an arbitrary number of objective functions. The Defining Point Algorithm is a generic scalarization-based approach that relies on the iterative solution of single-objective integer programs (IPs). It thus benefits from the strength of single-objective IP solvers. We provide a gentle introduction to the theoretical background and offer a tutorial on using an open source C++ implementation that utilizes CPLEX as the underlying (single-objective) IP solver. Together, we will solve exemplary instances of knapsack problems, assignment problems, and traveling salesman problems with three and more objective functions. There will also be room to discuss your own MOIP test problems. While the implementation aims to compute the complete nondominated set, the method can easily be adapted to compute approximations and/or representations of the Pareto front.

To prepare for the tutorial:

1. Install the C++ implementation of the Defining Point Algorithm available at:  
<https://github.com/kerstindaechert/DefiningPointAlgorithm>
2. The code requires the ILP-Solver CPLEX to solve subproblems. An academic license can be obtained free of charge:
  - (a) Register as an academic user at <https://www.ibm.com/academic/home>
  - (b) Select "Data Science"; then select "ILOG CPLEX Optimization Studio" and install the solver

## ***Notes***

## ***Notes***

## ***Notes***

Thanks  
for  
attending  
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SEE YOU NEXT TIME!

